

THE ASSIGNMENT PROBLEM

BASIC CONCEPTS AND FORMULA

Basic Concepts

1. The Assignment Algorithm

The Assignment Problem is another special case of LPP. It occurs when n jobs are to be assigned to n facilities on a one-to-one basis with a view to optimising the resource required.

2. Steps for Solving the Assignment Problem

Assignment problem can be solved by applying the following steps:

Step 1: Subtract the minimum element of each row from all the elements in that row. From each column of the matrix so obtained, subtract its minimum element. The resulting matrix is the starting matrix for the following procedure.

Step 2: Draw the minimum number of horizontal and vertical lines that cover all the zeros. If this number of lines is n , order of the matrix, optimal assignment can be made by skipping steps 3 and 4 and proceeding with step 5. If, however, this number is less than n , go to the next step.

Step 3: Here, we try to increase the number of zeros in the matrix. We select the smallest element out of these which do not lie on any line. Subtract this element from all such (uncovered) elements and add it to the elements which are placed at the intersections of the horizontal and vertical lines. Do not alter the elements through which only one line passes.

Step 4: Repeat steps 1, 2 and 3 until we get the minimum number of lines equal to n .

Step 5: (A) Starting with first row, examine all rows of matrix in step 2 or 4 in turn until a row containing exactly one zero is found. Surround this zero by, indication of an assignment there. Draw a vertical line through the column containing this zero. This eliminates any confusion of making any further assignments in that column. Process all the rows in this way.

(B) Apply the same treatment to columns also. Starting with the first column, examine all columns until a column containing exactly one zero is found. Mark and

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draw a horizontal line through the row containing this marked zero. Repeat steps 5A and B, until one of the following situations arises:

- (i) No unmarked () or uncovered (by a line) zero is left,
- (ii) There may be more than one unmarked zero in one column or row. In this case, put around one of the unmarked zero arbitrarily and pass 2 lines in the cells of the remaining zeros in its row and column. Repeat the process until no unmarked zero is left in the matrix.

3. Unbalanced Assignment Problems

Like the unbalanced transportation problems there could arise unbalanced assignment problems too. They are to be handled exactly in the same manner i.e., by introducing dummy jobs or dummy men, etc.

Question 1

An Electronic Data Processing (ED) centre has three expert Software professionals. The Centre wants three application software programs to be developed. The head of EDP Centre estimates the computer time in minutes required by the experts for development of Application Software Programs as follows:

Software programs	Computer time (in minutes) required by software Professionals
1	100 85 70
2	50 70 110
3	110 120 130

Assign the software professionals to the application software programs to ensure minimum usage of computer time.

Answer

The given problem is a balanced minimization assignment problem.

Step 1 & II: The minimum time elements in row 1, 2 and 3 are 70, 50 and 110 respectively. We subtract these elements from all elements in their respective row. The reduced matrix is shown in Table 1.

Table 1

Software Programs	A	B	C
1	30	15	0
2	0	20	60
3	0	10	20

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The minimum time elements in columns A, B and C are 0, 10, and 0 respectively. Subtract these elements from all the elements in their respective columns to get the reduced time matrix as shown in table 2.

Table 2

Software Programs	A	B	C
1	30	5	0
2	0	10	60
3	10	0	20

Step 3(a): The minimum number of horizontal and vertical lines to cover all zeros is 3, which is equal to the order of the matrix. Examine all rows one by one starting from row 1 until a row containing only single zero element is located. Assign this zero. All zero in the assigned column are crossed off as shown in table 3.

Table 3

Software Programs	A	B	C
1	30	5	0
2	0	10	60
3	10	0	20

Step 3(b): Now examine each column starting from A. There is only one zero in column. B Assign this cell as shown in table 4

Table 4

Software Programs	A	B	C
1	30	15	0
2	0	20	60
3	10	0	20

Step 3(c): Since the number of assignments (=3) equals the number of rows, the optimal solution is obtained. This Pattern of assignments among software professionals and programs with their respective time (in minutes) is given below:

Program	Software Professionals	Time (in Minutes)
1	C	70
2	A	50
3	B	120
	Total	240

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Question 2

A Production supervisor is considering, how he should assign five jobs that are to be performed, to five mechanists working under him. He wants to assign the jobs to the mechanists in such a manner that the aggregate cost to perform the jobs is the least. He has following information about the wages paid to the mechanists for performing these jobs:

Jobs

Mechanist	1	2	3	4	5
A	10	3	3	2	8
B	9	7	8	2	7
C	7	5	6	2	4
D	3	5	8	2	4
E	9	10	9	6	10

Assign the jobs to the mechanists so that the aggregate cost is the least.

Answer

The given problem is a standard minimization problem.

Subtracting minimum element of each row from all the elements of that row, the given problem reduces to

Jobs

Mechanist	1	2	3	4	5
A	8	1	1	0	6
B	7	5	6	0	5
C	5	3	4	0	2
D	1	3	6	0	2
E	3	4	3	0	4

Subtract the minimum element of each column from all the elements of that column. Draw the minimum number of lines horizontal or vertical so as to cover all zeros.

Jobs

Mechanist	1	2	3	4	5
A	7	0	0	0	4
B	6	4	5	0	3
C	4	2	3	0	0
D	0	2	5	0	0
E	2	3	2	0	2

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Since the minimum number of lines covering all zeros is equal to 4 which is less than the number of columns/rows (=5), the above table will not provide optimal solution. Subtract the minimum uncovered element (=2) from all uncovered elements and add to the elements lying on the intersection of two lines, we get the following matrix.

Jobs					
<i>Mechanist</i>	1	2	3	4	5
A	7	0	0	2	6
B	4	2	3	0	3
C	2	0	1	0	0
D	0	2	5	2	2
E	0	1	0	0	2

Since the minimum number of horizontal and vertical lines to cover all zeros is equal to five which is equal to the order of the matrix, the above table will give the optimal solution. The optimal assignment is made below:

Jobs					
<i>Mechanist</i>	1	2	3	4	5
A	7	2 0	0	2 0	6
B	4	2	3	0	3 0
C	2 0	0	1	0	0
D	0	2	5 0	2	2
E	0	1	0	0	2

The optimal assignment is given below:

Mechanist	Job	Wages
A	2	3
B	4	2
C	5	4
D	1	3
E	3	<u>9</u>
		<u>21</u>

The total least cost associated with the optimal mechanist-job assignment = 21

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Question 3

A project consists of four (4) major jobs, for which four (4) contractors have submitted tenders. The tender amounts, in thousands of rupees, are given below.

Jobs				
<i>Contractors</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	120	100	80	90
2	80	90	110	70
3	110	140	120	100
4	90	90	80	90

Find the assignment, which minimizes the total cost of the project. Each contractor has to be assigned one job.

Answer

The given problem is a standard minimization problem. Subtracting the minimum element of each row from all its elements in turn, the given problem reduces to

Jobs				
<i>Contractors</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	40	20	0	10
2	10	20	40	0
3	10	40	20	0
4	10	10	0	10

Now subtract the minimum element of each column from all its elements in turn. Draw the minimum number of lines horizontal or vertical so as to cover all zeros.

Jobs				
<i>Contractors</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	30	10	0	10
2	0	10	40	0
3	0	30	20	0
4	0	0	0	10

Since the minimum number of lines to cover all zeros is equal to 4(=order of the matrix), this matrix will give optimal solution. The optimal assignment is made in the matrix below:

The Assignment Problem

Contractors	Jobs			
	A	B	C	D
1	30	10	0	10
2	0	10	40	0
3	0	30	20	0
4	0	0	0	10

The optimal assignment is

Contractor	Job	Cost (in thousands of rupees)
1	C	80
2	A	80
3	D	100
4	B	90

Hence, total minimum cost of the project will be Rs.3,50,000.

Question 4

A project consists of four (4) major jobs, for which four (4) contractors have submitted tenders. The tender amounts, in thousands of rupees, are given below:

Contractors	Jobs			
	A	B	C	D
1	120	100	80	90
2	80	90	110	70
3	110	140	120	100
4	90	90	80	90

Final the assignment, which minimizes the total cost of the project. Each contractor has to be assigned one job.

Answer

The given problem is a standard minimization problem. Subtracting the minimum element of each row from all its elements in turn, the given problem reduces to

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<i>Contractors</i>	<i>Jobs</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	40	20	0	10
2	10	40	20	0
3	10	40	20	0
4	10	10	0	0

Now subtract the minimum element of each column from all its elements in turn. Draw the minimum number of lines horizontal or vertical so as to cover all zeros.

<i>Contractors</i>	<i>Jobs</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	30	40	0	10
2	0	10	40	0
3	0	30	20	0
4	0	0	0	10

Since the minimum number of lines to cover all zeros is equal to 4 (= order of the matrix), this matrix will give optimal solution. The optimal assignment is made in the matrix below.

<i>Contractors</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	30	40	0	10
2	0	10	40	0
3	0	30	20	0
4	0	0	0	10

The optimal assignment is:

Contractor	Job	Cost (in thousands of rupees)
1	C	80
2	A	80
3	D	100
4	B	90

Hence, total minimum cost of the project will be Rs. 3,50,000.

Question 5

A Marketing Manager has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each subordinate would take to perform each task is given in the matrix below. How should the task be allocated one to one man so that the total man-hours are minimised ?

	I	II	III	IV
1	16	52	34	22
2	26	56	8	52
3	76	38	36	30
4	38	52	48	20

Answer

	I	II	III	IV
1	16	52	34	22
2	26	56	8	52
3	76	38	36	30
4	38	52	48	20

Step 1:

Subtract the smallest element of each row from every element of the corresponding row

	I	II	III	IV
1	0	36	18	6
2	18	48	0	44
3	46	8	6	0
4	18	32	28	0

Step 2: Subtract the smallest element of each column from every element in that column

	I	II	III	IV
1	0	28	18	6
2	18	40	0	44
3	46	0	6	0
4	18	24	28	0

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Step 3: Drew minimum number of horizontal and vertical lines to cover all the zeros

	I	II	III	IV
1	0	28	18	6
2	18	40	0	44
3	46	0	6	0
4	18	24	28	0

The optimal assignment is

1 — I = 16

2 — III = 8

3 — II = 38

4 — IV = 20

82 hours

Minimum time taken = 82 hours

Question 6

A BPO company is taking bids for 4 routes in the city to ply pick-up and drop cabs. Four companies have made bids as detailed below:

Bids for Routes (Rs.)

Company/Routes	R_1	R_2	R_3	R_4
C_1	4,000	5,000	—	—
C_2	—	4,000	—	4,000
C_3	3,000	—	2,000	—
C_4	—	—	4,000	5,000

Each bidder can be assigned only one route. Determine the minimum cost that the BPO should incur.

Answer

Reducing minimum from each column element (figure in '000s)

Step 1					Step 2				
	R ₁	R ₂	R ₃	R ₄		R ₁	R ₂	R ₃	R ₄
C ₁	1	1	–	–	C ₁	0	0	–	–
C ₂	–	0	–	0	C ₂	–	0	–	0
C ₃	0	–	0	–	C ₃	0	–	0	–
C ₄	–	–	2	1	C ₄	–	–	1	0

Number of lines to connect all zeros nos. is 4 which is optional.

Alternatively you may also reduce the minimum from each row.

Step 1					Step 2				
	R ₁	R ₂	R ₃	R ₄		R ₁	R ₂	R ₃	R ₄
C ₁	0	1	–	–	C ₁	0	1	–	–
C ₂	–	0	–	0	C ₂	–	0	–	0
C ₃	1	–	0	–	C ₃	0	–	0	–
C ₄	–	–	0	1	C ₄	–	–	0	0

Number of lines to connect all zeros nos. is 4 which is optional.

All diagonal elements are zeros and are chosen. The minimum cost is Rs.15,000 C₁ – R₁ 4,000; C₂ – R₂ 4,000; C₃ – R₃ 2,000; C₄ – R₄ 5,000; (Total) = 15,000.

Question 7

A gear manufacturing company makes two types of gears – A and B. Both gears are processed on 3 machines, Hobbing M/c, Shaping M/c and Grinding M/c. The time required by each gear and total time available per week on each M/c is as follows:

	Gear (A)	Gear (B)	Available
Machine	(Hours)	(Hours)	Hours
Hobbing M/c	3	3	36
Shaping M/c	5	2	60
Grinding M/c	2	6	60
Other data:			
Selling price (Rs.)	820	960	
Variable cost (Rs.)	780	900	

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Determine the optimum production plan and the maximum contribution for the next week by simplex method. The initial table is given below:

		C_j	40	60	0	0	0
		Qty.					
C_j	Variable		X_1	X_2	X_3	X_4	X_5
0	X_3	36	3	3	1	0	0
0	X_4	60	5	2	0	1	0
0	X_5	60	2	6	0	0	1

Answer

Table 1

		C_j	40	60	0	0	0	Ratio	
		Qty							
C_j	Variable		X_1	X_2	X_3	X_4	X_5		
0	X_3	36	3	3	1	0	0	12	
0	X_4	60	5	2	0	1	0	30	
0	X_5	60	2	6	0	0	1	10	←
	Z_j	0	0	0	0	0	0		
	$Z_j - C_j$		-40	-60	0	0	0		

Table 2

		C_j	40	60	0	0	0	Ratio	
		Qty							
C_j	Variable		X_1	X_2	X_3	X_4	X_5		
0	X_3	6	2	0	1	0	$-\frac{1}{2}$	3	←
0	X_4	40	$\frac{13}{3}$	0	0	1	$-\frac{1}{3}$	$\frac{120}{13}$	
60	X_2	10	$\frac{1}{3}$	1	0	0	$\frac{1}{6}$	30	
	Z_j	600	20	60	0	0	10		
	$Z_j - C_j$		-20	0	0	0	10		

Table 3

		C_j Qty	40	60	0	0	0
c_j	Variable		X_1	X_2	X_3	X_4	X_5
40	X_1	3	1	0	$\frac{1}{2}$	0	$-\frac{1}{4}$
0	X_4	27	0	0	$-\frac{13}{6}$	1	$\frac{3}{4}$
60	X_2	9	0	1	$-\frac{1}{6}$	0	$\frac{1}{4}$
	Z_j	660	40	60	10	0	5
	$Z_j - C_j$		0	0	10	0	5

Since all $Z_j - C_j$ are positive or zero, this is the optimum solution with. $X_1 = 40$ and $X_2 = 60$ and optimum $Z = 660$.

Note: Alternatively, $C_j - Z_j$ may be used whereby maximum positive value may be considered.

Question 8

A company has four zones open and four marketing managers available for assignment. The zones are not equal in sales potentials. It is estimated that a typical marketing manager operating in each zone would bring in the following Annual sales:

Zones	Rs.
East	2,40,000
West	1,92,000
North	1,44,000
South	1,20,000

The four marketing managers are also different in ability. It is estimated that working under the same conditions, their yearly sales would be proportionately as under:

Manager M	:	8
Manager N	:	7
Manager O	:	5
Manager P	:	4

Required:

If the criterion is maximum expected total sales, find the optimum assignment and the maximum sales.

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Answer

Sum of the proportion = $(8 + 7 + 5 + 4) = 24$

Assuming Rs. 1,000 as one unit, the effective matrix is as follows:

Effective Matrix

Managers	Zones			
	East	West	North	South
M	$(8/24) \times 240 = 80$	$(8/24) \times 192 = 64$	$(8/24) \times 144 = 48$	$(8/24) \times 120 = 40$
N	$(7/24) \times 240 = 70$	$(7/24) \times 192 = 56$	$(7/24) \times 144 = 42$	$(7/24) \times 120 = 35$
O	$(5/24) \times 240 = 50$	$(5/24) \times 192 = 40$	$(5/24) \times 144 = 30$	$(5/24) \times 120 = 25$
P	$(4/24) \times 240 = 40$	$(4/24) \times 192 = 32$	$(4/24) \times 144 = 24$	$(4/24) \times 120 = 20$

Convert the maximization problem to minimization problem

The resultant loss matrix is as follows:

Loss Matrix

Managers	East	West	North	South
M	0	16	32	40
N	10	24	38	45
O	30	40	50	55
P	40	48	56	60

Row operation

Managers	East	West	North	South
M	0	16	32	40
N	0	14	28	35
O	0	10	20	25
P	0	8	16	20

Column operation

Managers	East	West	North	South
M	0	8	16	20
N	0	6	12	15
O	0	2	4	5
P	0	0	0	0

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Managers	East	West	North	South
M	0	6	14	18
N	0	4	10	13
O	0	0	2	3
P	2	0	0	0

Managers	East	West	North	South
M	0	2	10	14
N	0	0	6	9
O	4	0	2	3
P	6	0	0	0

Managers	East	West	North	South
M	0	2	8	12
N	0	0	4	7
O	4	0	0	1
P	8	2	0	0

Assignment

Sales

Rs.

M – East 80,000

N – West 56,000

O – North 30,000

P – South 20,000

1,86,000

Question 9

The cost matrix giving selling costs per unit of a product by salesman A, B, C and D in regions R_1 , R_2 , R_3 and R_4 is given below:

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	A	B	C	D
R_1	4	12	16	8
R_2	20	28	32	24
R_3	36	44	48	40
R_4	52	60	64	56

- (i) Assign one salesman to one region to minimise the selling cost.
- (ii) If the selling price of the product is Rs. 200 per unit and variable cost excluding the selling cost given in the table is Rs. 100 per unit, find the assignment that would maximise the contribution.
- (iii) What other conclusion can you make from the above?

Answer

(i)

4	12	16	8
20	28	32	24
36	44	48	40
52	60	64	56

Subtracting minimum element – each row.

0	8	12	4
0	8	12	4
0	8	12	4
0	8	12	4

Subtracting minimum element – each column,

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Minimum no. of lines to cover all zeros = 4 = order of matrix. Hence optimal assignment is possible.

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Minimum cost $= 4 + 28 + 48 + 56 = 136.$

$$= AR_1 + BR_2 + CR_3 + DR_4$$

Since all are zeros, there are 24 solutions to this assignment problem.

Viz.	A	B	C	D
	R ₁	R ₂	R ₃	R ₄
	R ₂	R ₃	R ₄	R ₁
	R ₃	R ₄	R ₁	R ₂
	R ₄	R ₁	R ₂	R ₃
	R ₁	R ₃	R ₄	R ₂ etc.

A can be assigned in 4 ways, B in 3 ways for each of A's 4 ways.

(ii) SP – VC = 100 Rs.

	A	B	C	D
R ₁	96	88	84	92
R ₂	80	72	68	76
R ₃	64	56	52	60
R ₄	48	40	36	44

Subtracting the highest term

0	8	12	4
16	24	28	20
32	40	44	36
48	56	60	52

Subtracting minimum term of each row.

0	8	12	4
0	8	12	4
0	8	12	4
0	8	12	4

Which is the same as the earlier matrix

Maximum contribution = Rs. (96 + 72 + 52 + 44) = Rs. 264.

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Alternative Solution:

Maximisation of contribution is same as minimizing cost. Hence, same assignments as in (i) will be the optional solution.

Maximum Contribution Rs. $(400 - 136) = \text{Rs. } 264$

- (iii) (a) The relative cost of assigning person i to region r does not change by addition or subtraction of a constant from either a row, or column or all elements of the matrix.
- (b) Minimising cost is the same as maximizing contribution. Hence, the assignment solution will be the same, applying point (i) above.
- (c) Many zero's represent many feasible least cost assignment. Here, all zeros mean maximum permutation of a 4×4 matrix, viz. $4 \times 3 \times 2 \times 1 = 24$ solutions are possible.

Question 10

In an assignment problem to assign jobs to men to minimize the time taken, suppose that one man does not know how to do a particular job, how will you eliminate this allocation from the solution?

Answer

In an assignment minimization problem, if one task cannot be assigned to one person, introduce a prohibitively large cost for that allocation, say M , where M has a high the value. Then, while doing the row minimum and column minimum operations, automatically this allocation will get eliminated.

Question 11

A factory is going to modify of a plant layout to install four new machines $M1$, $M2$, $M3$ and $M4$. There are 5 vacant places J , K , L , M and N available. Because of limited space machine $M2$ cannot be placed at L and $M3$ cannot be placed at J . The cost of locating machine to place in Rupees is shown below:

	(Rs.)				
	J	K	L	M	N
$M1$	18	22	30	20	22
$M2$	24	18	--	20	18
$M3$	--	22	28	22	14
$M4$	28	16	24	14	16

The Assignment Problem

Required:

Determine the optimal assignment schedule in such a manner that the total costs are kept at a minimum.

Answer

Dummy machine (M5) is inserted to make it a balanced cost matrix and assume its installation cost to be zero. Cost of install at cell M3 (J) and M2 (L) is very high marked as é.

	J	K	L	M	N
M1	18	22	30	20	22
M2	24	18	é	20	18
M3	é	22	28	22	14
M4	28	16	24	14	16
M5 (Dummy)	0	0	0	0	0

Step 1

Subtract the minimum element of each row from each element of that row

	J	K	L	M	N
M1	0	4	12	2	4
M2	6	0	é	2	0
M3	é	8	14	8	0
M4	14	2	10	0	2
M5 (Dummy)	0	0	0	0	0

Step 2

Subtract the minimum element of each column from each element of that column

	J	K	L	M	N
M1	0	4	12	2	4
M2	6	0	é	2	0
M3	é	8	14	8	0
M4	14	2	10	0	2
M5 (Dummy)	0	0	0	0	0

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Step 3

Draw lines to connect the zeros as under:

	J	K	L	M	N
M1	0	4	12	2	4
M2	6	0	e	2	0
M3	e	8	14	8	0
M4	14	2	10	0	2
M5 (Dummy)	0	0	0	0	0

There are five lines which are equal to the order of the matrix. Hence the solution is optimal. We may proceed to make the assignment as under:

	J	K	L	M	N
M1	0	4	12	2	4
M2	6	0	e	2	0
M3	e	8	14	8	0
M4	14	2	10	0	2
M5 (Dummy)	0	0	0	0	0

The following is the assignment which keeps the total cost at minimum:

Machines	Location	Costs Rs.
M1	J	18
M2	K	18
M3	N	14
M4	M	14
M5 (Dummy)	L	0
Total		64

EXERCISE

Question 1

A Car hiring company has one car at each of the five depots A,B,C,D and E. A customer in each of the five towns V,W,X,Y and requires a car. The distance in kms, between depots (origin) and the towns (destination) are given in the following table:

		Depots				
		A	B	C	D	E
Towns	V	3	5	10	15	8
	W	4	7	15	18	8
	X	8	12	20	20	12
	Y	5	5	8	10	6
	Z	10	10	15	25	10

Find out as to which car should be assigned to which customer so that the total distance traveled is a minimum. How much is the total traveled distance?

Answer

The optimal assignment is

Town	Depot	Distance (in kms)
V	C	10
W	B	7
X	A	8
Y	D	10
Z	E	<u>10</u>
	Total	<u>45</u>

Hence the minimum total traveled distance = 45 kms.

Question 2

ABC airline operating 7 days a week has given the following time-table. Crews must have minimum layover of 5 hours between flights. Obtain the pairing flights that minimize the layover time away from home. For any given pairing the crew will be based at the city that results in the smaller layover.

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Hyderabad-Delhi			Delhi-Hyderabad		
Flight No.	Depart.	Arrive	Flight No.	Depart.	Arrive
A1	6 AM	8 AM	B1	8 AM	10 AM
A2	8 AM	10 AM	B2	9 AM	11 AM
A3	2 PM	4 PM	B3	2 PM	4 PM
A4	8 PM	10 PM	B4	7 PM	9 PM

Answer

The optimal assignment is

From Flight No.	To Flight No.	Layover time
A ₁	B ₃	6
A ₂	B ₄	9
A ₃	B ₁	16
A ₄	B ₂ *	9
		40 hours

Question 3

Solve the assignment problem represented by the following effective matrix:

	a	b	c	d	e	f
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	26	11	57	22	25	18
F	3	56	53	31	17	28

Answer

The assignment is

- (i) A→d, B→f, C→b, D→c, E→e, F→a

And total effect = 11+48+28+27+25+3=142

Alternate solutions exist. One of the alternate solutions is given by

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(ii) $A \rightarrow d, B \rightarrow a, C \rightarrow f, D \rightarrow c, E \rightarrow b$ and $F \rightarrow e$ with total effect = 142

Question 4

To stimulate interest and provide an atmosphere for intellectual discussion, a finance faculty in a management school decides to hold special seminars on four contemporary topics: leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once in a week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

	Leasing	Portfolio Management	Private Mutual Fund	Swaps & Options
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	60	20	30	20
Thursday	30	30	20	30
Friday	10	20	10	30

Find an optimal schedule of the seminars. Also find out the total number of students who will be missing at least one seminar.

Answer

And the optimal schedule is

			No. of Students Missing
Monday	:	Swaps and options	20
Tuesday	:	No Seminar	0
Wednesday	:	Portfolio Management	20
Thursday	:	Pvt. Mutual funds	20
Friday	:	Leasing	<u>10</u>
			<u>70</u>

Thus, the total number of students who will be missing at least one seminar = 70

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Question 5

A manufacturing company has four zones A, B, C, D and four sales engineers P, Q, R, S respectively for assignment. Since the zones are not equally rich in sales potential, therefore it is estimated that a particular engineer operating in a particular zone will bring the following sales;

Zone A	:	4,20,000
Zone B	:	3,36,000
Zone C	:	2,94,000
Zone D	:	4,62,000

The engineers are having different sales ability. Working under the same conditions, their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineer to the richest zone, the next best to the second richest zone and so on.

Find the optimum assignment and the maximum sales.

Answer

The optimum assignments are as follows:

Zones (Loss in thousands of rupees)				
Sales Engineer	A	B	C	D
P	3	13	19	0
Q	0	0	1	2
R	0	4	7	0
S	2	0	0	5

Engineers	Zones	Sales (in Rs.)
P	D	1,54,000
Q	B	72,000
R	A	1,10,000
S	C	<u>56,000</u>
		<u>3,92,000</u>

It can be seen from the above assignments that the best engineer P is assigned to the richest Zone D, the next best engineer R is assigned to second richest zone A, the next

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best engineer Q is assigned to zone B and so on. Hence, the optimum assignment matches the company's criteria of achieving the maximum expected total sales.

Question 6

An organization is producing 4 different products viz. A, B, C, and D having 4 operators viz. P, Q, R and S, who are capable of producing any of the four products, works effectively 7 hours a day. The time (in minutes) required for each operator for producing each of the product are given in the cells of the following matrix along profit (Rs. per unit):

Operator	Product			
	A	B	C	D
P	6	10	14	12
Q	7	5	3	4
R	6	7	10	10
S	20	10	15	15
Profit (Rs./Units)	3	2	4	1

Find out the assignment of operators to products which will maximize the profit.

Answer

Specific assignments in this case are as below:

Operator	Product	Profit (Rs.)
P	A	210
Q	C	560
R	B	120
S	D	28
Total	Profit (Rs.)	918

Question 7

A private firm employs typists on hourly piece rate basis for their daily work. Five typists are working in that firm and their charges and speeds are different. On the basis of some earlier understanding, only one job is given to one typist is paid for full hours even when he or she works for a fraction of an hour. Find the least cost allocation for the following when he or she works for a fraction of an hour. Find the least cost allocation for the following data:

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<i>Typist</i>	<i>Rate per hour (Rs.)</i>	<i>Number of pages typed hour</i>	<i>Job</i>	<i>No. of pages</i>
<i>A</i>	<i>5</i>	<i>12</i>	<i>P</i>	<i>199</i>
<i>B</i>	<i>6</i>	<i>14</i>	<i>Q</i>	<i>175</i>
<i>C</i>	<i>3</i>	<i>8</i>	<i>R</i>	<i>143</i>
<i>D</i>	<i>4</i>	<i>10</i>	<i>S</i>	<i>298</i>
<i>E</i>	<i>4</i>	<i>11</i>	<i>T</i>	<i>178</i>

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Answer

Cost (Rs.)

Thus typist A is given job	T	75
Thus typist B is given job	R	66
Thus typist C is given job	Q	66
Thus typist D is given job	P	80
Thus typist E is given job	S	112
	Total	Rs.399

Note: In case the above solution is not unique. Alternate solution also exists.

Question 8

XYZ airline operating 7 days a week has given the following time-table. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing flights and minimizes layover time away from home. For any given pairing the crew will be based at the city that results in the smaller layover:

<i>Chennai Mumbai</i>			<i>Mumbai - Chennai</i>		
<i>Flight Number</i>	<i>Depart.</i>	<i>Arrive</i>	<i>Flight Number</i>	<i>Depart.</i>	<i>Arrive</i>
<i>A1</i>	<i>6 AM</i>	<i>8 AM</i>	<i>B1</i>	<i>8 AM</i>	<i>10 AM</i>
<i>A2</i>	<i>8 AM</i>	<i>10 AM</i>	<i>B2</i>	<i>9 AM</i>	<i>11 AM</i>
<i>A3</i>	<i>2 PM</i>	<i>4 PM</i>	<i>B3</i>	<i>2 PM</i>	<i>4 PM</i>
<i>A4</i>	<i>8 PM</i>	<i>10 PM</i>	<i>B4</i>	<i>7 PM</i>	<i>9 PM</i>

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Answer

The optimal assignment is

<i>From Flight No.</i>	<i>To Flight No.</i>	<i>Layover</i>
A1	B3	6
A2	B4	9
A3	B1	16
A4	B2*	9
		40 hours

Question 9

A firm produces four products.. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allows 30 minutes for lunch. The processing time in minutes and the profit for each of the products are given below:

<i>Operators</i>	<i>Products</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	15	9	10	6
2	10	6	9	6
3	25	15	15	9
4	15	9	10	10
<i>Profit (Rs.) p. u.</i>	8	6	5	4

Find the optimal assignment of products to operators.

Answer

The optimal assignment is as shown below:

<i>Operator</i>	<i>Product</i>	<i>Profit (Rs.)</i>
1	D	300
2	B	450
3	C	150
4	A	240
	Rs.	1140